Influence of Cavity BPM resolution due to high bunch repetition rate

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Cavity BPMs are the most promising beam position monitors to detect the beam position with best resolution. Proposed electron accelerators with high duty cycle could cause that following bunches add a signal to the previous signals. This can increase the resolution under special conditions. Here the contribution due to arrival time jitter is derived and examples are given. The contribution for few bunches is small and is minimized when the phase offset between the signals of following bunches are zero or 180°. For a high number of bunches the phase offset of 180° is preferred because at this condition the influence due to arrival time drifts is minimized.

I. INTRODUCTION

Future linear electron accelerators are designed to produce bunches with high repetition rates within one pulse train to increase the duty cycle for high efficiency. The proposed bunch spacing are 0.5 ns [1], 50 ns [2] and above 180 ns [3, 4]. Each machine aims to produce small transverse beam sizes, either for collision or for FEL operation. Therefore they need a beam position detection with high resolution. Cavity Beam Position Monitors (BPM) are the most promising device to detect the beam position with best resolution. A beam induces modes in the cavity such that the position can be detected by analyzing the oscillating signal. With short bunch spacing the oscillation could not be vanished when the next bunch arrives the BPM. For single bunch measurement each position has to be calculated with respect to the previous bunches. This could reduce the position resolution for the following bunches in one train. Here the resolution influence for one bunch is estimated based on the influence of both cavities (dipole and reference cavities) followed by the calculation of the resolution change due to following bunches.

II. POSITION RESOLUTION OF A CAVITY BPM

A. Single bunch

When a charge particle traverses a cavity it induces electro-magnetic modes. The amplitude of a dipole mode with resonance frequency f_0 is a function of the offset. Therefore by analyzing the amplitude the offset can be detected, for a detailed description see [5]. The resolution of the voltage amplitude ΔU depends on the beam angle, beam tilt, common mode leakage and thermal noise, see [6]. The amplitude of the dipole mode U_D has to be normalized with a reference amplitude U_R, because it is a function of the beam charge q too. This is mostly done by using a reference cavity designed for the monopole mode with the same resonance frequency f_0; the phase difference between both cavity signals provides the direction of the offset. By using an overall calibration k the offset x can be calculated by using peak detection with

$$x = k\frac{U_D}{U_R}. \tag{1}$$

The position resolution Δx depends on the resolution of the measured amplitudes ΔU_D and ΔU_R. Thus results in

$$\begin{aligned}\Delta x &= \sqrt{\left(\frac{\delta x}{\delta U_D}\Delta U_D\right)^2 + \left(\frac{\delta x}{\delta U_R}\Delta U_R\right)^2} \\ &= \frac{k}{U_R}\sqrt{\Delta U_D^2 + \left(\frac{U_D}{U_R}\Delta U_R\right)^2}.\end{aligned} \tag{2}$$

Both amplitudes have different levels ($U_R \gg U_D$) but due to the same frequency mostly processed with the same kind of electronics. Therefore the resolution of the amplitudes are similar: $\Delta U \sim \Delta U_D \sim \Delta U_R$. This simplifies equation (2) to

$$\Delta x = \frac{k\Delta U}{U_R}\sqrt{1 + \left(\frac{U_D}{U_R}\right)^2}. \tag{3}$$

The first term indicates the typical behavior of a BPM resolution to the beam charge because $q \sim U_R$. The resolution for a fixed charge becomes the best when the offset is small because U_D is small. For higher offset the resolution becomes more worse. But the amplitude of the monopole mode is much higher than the dipole mode thus the second term in the square root is much smaller than 1. Therefore the dependency of the resolution on the beam offset is weak. This is important for the BPM resolution with high repetition rates because next bunches will add a signal to the present amplitude but the amplitude resolution does not change significantly.

B. Two bunches

Lets write the single signal as a function of time

$$U_1(t) = U_D e^{-\frac{t}{\tau}}\cos(\omega t), \tag{4}$$

where the decay time $\tau = Q_L/(\pi f_0)$ with Q_L the loaded quality factor of the cavity and $\omega = 2\pi f_0$. Two bunches separated by the bunch spacing t_B generates a signal corresponding to

$$U_2(t) = U_1(t) + U_1(t - t_B), \tag{5}$$

for $t \geq t_B$; the bunch repetition rate is $1/t_B$. At the time t_B the amplitude will be detected and thus

$$U_2(t_B) = U_D \left[e^{-\frac{t_B}{\tau}} \cos(\omega t_B) + 1 \right]. \tag{6}$$

The last equation indicates an additional contribution to the resolution due to the error of the bunch spacing Δt_B, the time jitter. For large bunch spacing the additional term is negligible because the exponential term is already zero.

The additional contribution to the resolution can be written as

$$\Delta U_2^2 = \left(\frac{\delta U_2(t_B)}{\delta t_B} \right)^2 \Delta t_B^2. \tag{7}$$

Using equation (6) the derivative is

$$\frac{1}{U_D} \frac{\delta U_2(t_B)}{\delta t_B} = -e^{-\frac{t_B}{\tau}} \left[\frac{1}{\tau} \cos(\omega t_B) + \omega \sin(\omega t_B) \right]. \tag{8}$$

Typical value of the quality factor is 1000 with a resonance frequency at about 5 GHz. This results in a decay time of 64 ns, or $1/\tau \sim 0.016$ GHz, whereas $\omega \sim 31$ GHz. Therefore the sin term in equation (8) dominates. The additional contribution to the resolution vanishes when the bunch spacing and the resonance frequency is $n\pi = \omega t_B$ with n an integer value. This is realized when the bunch spacing is

$$t_B = \frac{n}{2 f_0}. \tag{9}$$

This means that the following bunch produces a negligible resolution contribution when the amplitude is added or subtracted from the previous bunch signal; or the bunch arrives the cavity when the previous signal has a phase of zero or 180° to the new signal. The worst case is at 90° and 270°.

C. Three bunches

Equation (5) can be extrapolated to three bunches to

$$U_3(t) = U_1(t) + U_1(t - t_B) + U_1(t - 2t_B) \tag{10}$$

for $t \geq 2t_B$. For peak detection the amplitude becomes

$$U_3(2t_B) = U_D \left[e^{-\frac{2t_B}{\tau}} \cos(2\omega t_B) + e^{-\frac{t_B}{\tau}} \cos(\omega t_B) + 1 \right]. \tag{11}$$

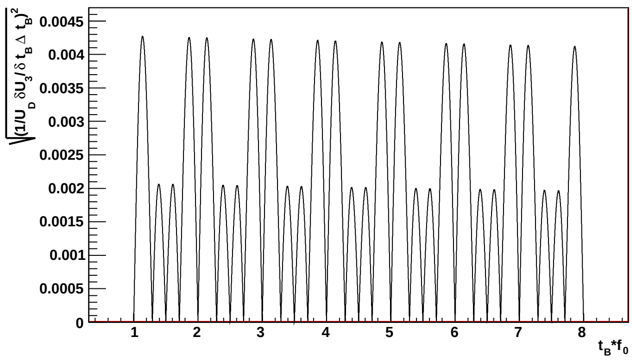

FIG. 1. Contribution to the position resolution by the bunch arrival time jitter of 50 fs for three bunches as a function of bunch spacing and resonance frequency up to 8 revolutions.

The additional contribution to the position resolution is similar to equation (7), replacing the 2 with a 3. The derivative of U_3 to t_B is

$$\frac{1}{U_D} \frac{\delta U_3(2t_B)}{\delta t_B} = -e^{-\frac{2t_B}{\tau}} \left[\frac{2}{\tau} \cos(2\omega t_B) + 2\omega \sin(2\omega t_B) \right] - e^{-\frac{t_B}{\tau}} \left[\frac{1}{\tau} \cos(\omega t_B) + \omega \sin(\omega t_B) \right]. \tag{12}$$

By neglecting the *cos* terms one can see that an additional contribution appears with twice of the oscillation. But the statement of equation (9) still holds: the contribution is vanishing for phases of zero and 180°.

In Figure 1 the additional contribution to the position resolution is shown as a function of $t_B f_0$; integer number means a phase offset of zero, half integer number implies a phase offset of 180°. Here a typical bunch arrival time jitter of $\Delta t_B = 50$ fs is used. It is obvious the minimized contribution at integer and half integer numbers of the resonance frequency. In addition the doubled oscillation contributes with lower amplitude to the resolution because of the lower amplitude due to the smaller $exp(-2t_B/\tau)$ term. The envelope decays with the exponential term according the decay time. The *cos*-terms in equation (12) are included in the Figure 1 but not visible because their contributions are negligible. The overall amplitude of the additional contribution is still much lower than one, therefore the contribution due to the time jitter is negligible with the assumed parameters. When the arrival time jitter exceeds 12 ps the contribution is not negligible any more.

D. N-bunches

The signal for N-bunches can be written as

$$U_N(t) = \sum_{n=0}^{N-1} U_1(t - n t_B) \tag{13}$$

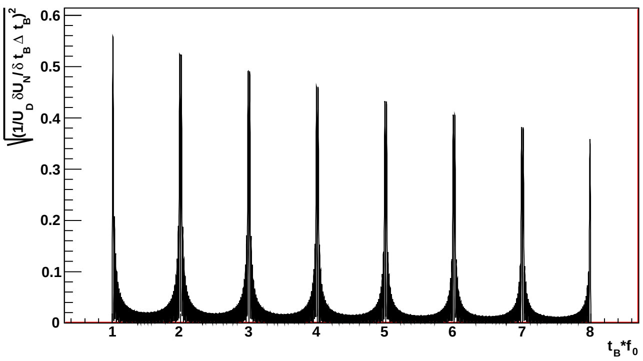

FIG. 2. Contribution to the position resolution by the bunch arrival jitter of 50 fs for 30 bunches as a function of bunch spacing and resonance frequency up to 8 revolutions.

for $t \geq (N-1)t_B$. The amplitude is, when the Nth bunch arrives the BPM,

$$U_N((N-1)t_B) = \sum_{n=0}^{N-1} U_1((N-1)t_B - nt_B). \tag{14}$$

The additional contribution to the position resolution becomes

$$\Delta U_N^2 = \left(\frac{\delta U_N((N-1)t_B)}{\delta t_B}\right)^2 \Delta t_B^2. \tag{15}$$

The derivative can be expressed like

$$\frac{\delta U_N((N-1)t_B)}{\delta t_B} = \sum_{n=0}^{N-1} \frac{\delta U_1((N-1-n)t_B)}{\delta t_B}. \tag{16}$$

Each summand evolves to

$$\frac{1}{U_D}\frac{\delta U_1((N-1-n)t_B)}{\delta t_B} = -e^{-\frac{(N-1-n)t_B}{\tau}}(N-1-n)\cdot \left[\frac{1}{\tau}\cos\left((N-1-n)\omega t_B\right) + \omega \sin\left((N-1-n)\omega t_B\right)\right]. \tag{17}$$

The equations derived in this subsection are a more general approach compared to the both cases before.

In Figure 2 the additional contribution according equation (15) with respect of the equations (16) and (17) is shown for 30 bunches. The envelope is decaying faster compared to the 3 bunches case. But the amplitude is much higher for the first revolutions. Still the contribution is minimized for phases of zero and 180°, see zoomed Figure 3. But a time drift could increase the contribution significantly when $t_B f_0$ is around an integer. For the present example this would maximize the contribution for drifts of 2 ps; this could be possible when different bunch compression before the BPM will be used. Therefore it would be more save to design a cavity BPM where

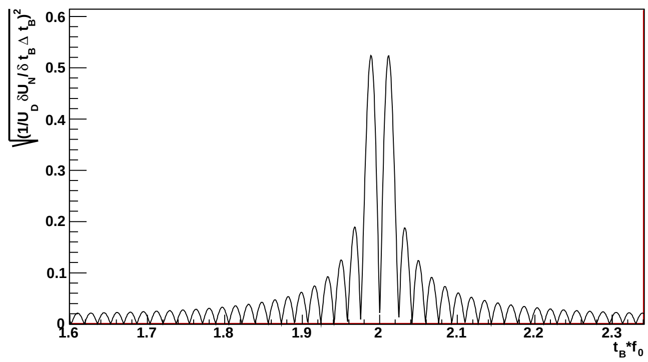

FIG. 3. Zoom of Figure 2 around $t_B f_0 = 2$.

the bunch signals would be added by 180° because the influence of drifts to the resolution is minimized.

For higher number of bunches in one train the amplitudes around the integer values are increasing. The envelope amplitudes at half integer numbers is still much lower than 1. Therefore a half integer value or a phase offset of 180° for a cavity BPM design is preferred.

III. SUMMARY

In this study the additional contribution of following bunches to the position resolution of cavity BPMs is calculated. It is shown that the contribution is minimized when the phase difference between previous and new bunch is zero or 180°. The amplitude for few bunches is still negligible when the bunch arrival time jitter is below few ps. But for several bunches the stability against drifts is better for phases of 180°.

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