Technical Note
2007-01
January 2007

Sensitivity Estimation for the PETRA-III Beam Position Monitors Based on a Boundary Element Method

G. Kube

Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany

Sensitivity Estimation for the PETRA-III Beam Position Monitors based on a Boundary Element Method

G. Kube
- MDI -

January 29, 2007

Abstract

For measurement and control of the PETRA-III closed orbit it is planned to use electrostatic beam position monitors (pickups). Due to the strong asymmetry in the ring geometry there exist several vacuum chamber cross–sections, and as consequence five monitor configurations will be installed. The position sensitivity for these monitor configurations has been calculated with a boundary element method (BEM) and the results are comprised in this note.

1 Introduction

PETRA-III will be a new high-brilliance synchrotron radiation source on the DESY site in Hamburg-Bahrenfeld. From 2007 on, the PETRA accelerator at the Helmholtz research center DESY will be converted into the most brilliant storage–ring–based X–ray source worldwide. Nearly 300 meters of the 2.3 km long storage ring have to be rebuilt completely to convert PETRA into a powerful light source. For measurement and control of the PETRA-III closed orbit it is planned to install about 220 electrostatic beam position monitors (BPMs): one BPM per standard FODO cell and additional BPMs at the locations of insertion devices. Due to the unconventional asymmetry in the ring geometry there exist different vacuum sections with varying cross–section. Some of the BPM pickup stations will be installed in non–standard vacuum chambers. All BPMs are used for beam orbit observation and are also included in the orbit stabilization feedback system. In order to fulfill the required position resolution and to reconstruct accurately the beam transverse position in the μm level and even better, the position sensitivity of each type of electrostatic pickup has to be analyzed. This note deals with theoretical calculations of the pickup properties based on a numerical solution of the integral representation for the scalar potential. For this purpose a boundary element method (BEM) described in Ref. [1] was utilized.

2 Theory

In order to calculate the position sensitivity of an electrostatic BPM it is necessary to determine the charge induced onto the button electrodes which are arranged symmetrically in the vacuum chamber profile. The determination of the beam induced signals onto the button electrodes can be reduced to a two–dimensional electrostatic problem in case of relativistic beams with the induced charge on the boundary as unknown [2]. The task is to solve this electrostatic problem in a closed region surrounded by the beam pipe which determines the boundary condition.

The basic equation describing this kind of problem is the Poisson equation relating the scalar potential $\phi(\vec{r})$ and the charge density $\rho(\vec{r})$, representing the beam charge:

$$\vec{\nabla}^2\phi(\vec{r}) = -\frac{\rho(\vec{r})}{\varepsilon_0} \quad . \tag{1}$$

In order to find a solution for the scalar potential consider Green's theorem for two scalar fields Φ, Ψ:

$$\int_V [\Phi\vec{\nabla}^2\Psi - \Psi\vec{\nabla}^2\Phi]\mathrm{d}^3r = \oint_S [\Phi\frac{\partial\Psi}{\partial n} - \Psi\frac{\partial\Phi}{\partial n}]\mathrm{d}^2r \tag{2}$$

with V an arbitrary three–dimensional volume surrounded by a closed surface S. A special choice for the scalar fields is

$$\Phi = \phi(\vec{r}), \qquad \Psi = G(\vec{r}, \vec{r'}) \quad , \tag{3}$$

with $\phi(\vec{r})$ the scalar potential described by Eq.(1) and $G(\vec{r})$ the related Greens function, satisfying

$$\vec{\nabla}^2 G(\vec{r}, \vec{r'}) = -\delta(\vec{r} - \vec{r'}) \quad . \tag{4}$$

Inserting Eq.(1) and Eq.(4) in Eq.(2) results in an integral representation for the scalar potential:

$$\phi(\vec{r}) = \frac{1}{\varepsilon_0} \int_V G(\vec{r}, \vec{r'}) \rho(\vec{r'}) \mathrm{d}^3 r + \oint_S [G(\vec{r}, \vec{r'}) \frac{\partial \phi(\vec{r'})}{\partial n} - \phi(\vec{r'}) \frac{\partial G(\vec{r}, \vec{r'})}{\partial n}] \mathrm{d}^2 r \quad . \tag{5}$$

The derivative of ϕ with respect to the surface normal $\hat{n}$ can be expressed as

$$\frac{\partial \phi(\vec{r})}{\partial n} = \vec{\nabla} \phi(\vec{r}) \cdot \hat{n} = -\vec{E} \cdot \hat{n} = \frac{\sigma(\vec{r})}{\varepsilon_0} \quad . \tag{6}$$

In addition, assuming perfect conductivity on the surface S (i.e. the beam pipe), the scalar potential satisfies a Dirichlet–type boundary condition

$$\phi(\vec{r}) = 0 \qquad \text{for} \qquad \vec{r} \,\epsilon\, S. \tag{7}$$

As consequence it is described by the following integral representation

$$\phi(\vec{r}) = \frac{1}{\varepsilon_0} \int_V G(\vec{r}, \vec{r'}) \rho(\vec{r'}) \, \mathrm{d}^3 r + \frac{1}{\varepsilon_0} \oint_S G(\vec{r}, \vec{r'}) \sigma(\vec{r'}) \, \mathrm{d}^2 r \quad . \tag{8}$$

In the case of a two–dimensional electrostatic problem the Greens function can be determined by solving Eq.(4) directly, leading to

$$G(\vec{r}, \vec{r'}) = -\frac{1}{2\pi} \ln |\vec{r} - \vec{r'}| = \frac{1}{2\pi} \ln \frac{1}{|\vec{r} - \vec{r'}|} \; . \tag{9}$$

Following Ref. [3] the two–dimensional region containing the charge density $\rho(x, y)$ is denoted by

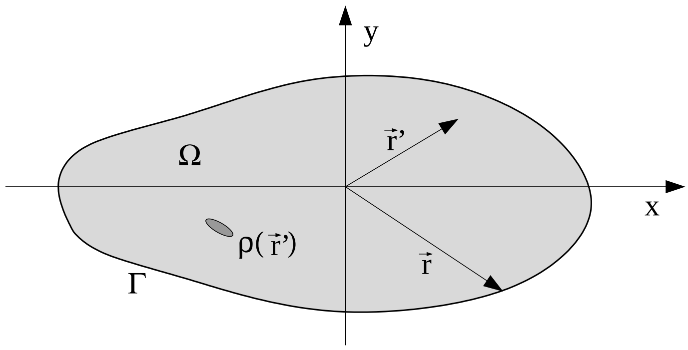

Figure 1: Sketch of the two–dimensional electrostatic problem. The area Ω is surrounded by the closed curve Γ (i.e. the vacuum chamber boundary). The charge density ρ describes the beam.

Ω and the boundary (beam pipe) by Γ, cf. Fig.1. As result the integral form describing the solution of the electrostatic problem is determined by the following equation:

$$\phi(\vec{r}) = \frac{1}{2\pi\varepsilon_0} \int_\Omega \rho(\vec{r'}) \ln \frac{1}{|\vec{r} - \vec{r'}|} \, \mathrm{d}\Omega + \frac{1}{2\pi\varepsilon_0} \oint_\Gamma \sigma(\vec{r'}) \ln \frac{1}{|\vec{r} - \vec{r'}|} \, \mathrm{d}\Gamma \quad . \tag{10}$$

An additional simplification arises if the transverse beam dimensions are considered to be infinitesimal small, i.e. the charge distribution is described by

$$\rho(\vec{r'}) = \rho_0 \, \delta(\vec{r}_0 - \vec{r'}) \, . \tag{11}$$

In this case the integration over the area Ω can be carried out directly, leading to

$$\phi(\vec{r}) = \frac{\rho_0}{2\pi\varepsilon_0}\ \ln\frac{1}{|\vec{r}-\vec{r}_0|} + \frac{1}{2\pi\varepsilon_0}\oint_\Gamma \sigma(\vec{r'})\ \ln\frac{1}{|\vec{r}-\vec{r'}|}\ \mathrm{d}\Gamma \quad . \tag{12}$$

This equation is the basis for all further calculations.

Except for special cases Eq.(12) has to be solved numerically. According to Ref. [1] this can be done accurately using a boundary element method. For this purpose the boundary is approximated by small line segments as shown in Fig. 2.

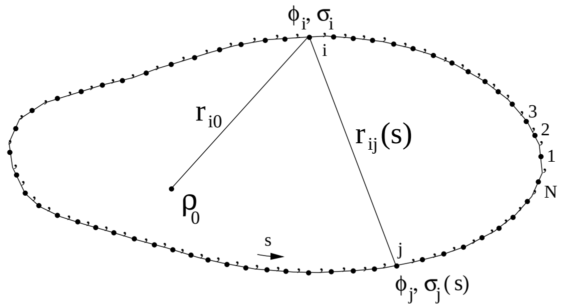

Figure 2: Boundary element method: discretization of the boundary Γ by a finite number N of small line segments.

The potential for the ith element is given by

$$\phi_i = \frac{\rho_0}{2\pi\varepsilon_0}\ \ln\frac{1}{r_{i0}} + \frac{1}{2\pi\varepsilon_0}\sum_{j=1}^{N}\int_{\Gamma_j}\sigma_j(s)\ \ln\frac{1}{r_{ij}(s)}\ \mathrm{d}s \quad . \tag{13}$$

Assuming the boundary element small enough the charge density $\sigma_j(s)$ is approximated to be constant within this element, and Eq.(13) becomes

$$\phi_i = \frac{\rho_0}{2\pi\varepsilon_0}\, G_{i0} + \frac{\sigma_j}{2\pi\varepsilon_0}\sum_{j=1}^{N} G_{ij} \tag{14}$$

with the notation

$$G_{i0} = \ln\frac{1}{r_{i0}} \quad , \tag{15}$$

$$G_{ij} = \int_{\Gamma_j}\ln\frac{1}{r_{ij}(s)}\ \mathrm{d}s \tag{16}$$

according to Ref. [1].

The boundary Γ consists of N boundary elements Γ_j. Therefore Eq.(14) represents a matrix equation

$$[\phi_i] = \frac{\rho_0}{2\pi\varepsilon_0}\,[G_{i0}] + \frac{1}{2\pi\varepsilon_0}\,[G_{ij}]\,[\sigma_j] \tag{17}$$

with $[\phi_i], [G_{i0}]$ column vectors of length N and $[G_{ij}]$ an $N\times N$ matrix.

In the present case with Γ the boundary of a beam pipe the potential of the boundary is zero. Hence by matrix inversion Eq.(17) can simply be rewritten as

$$[\sigma_j] = -\rho_0[G_{ij}]^{-1}[G_{i0}]\,, \tag{18}$$

and the induced charge density in the beam pipe can be calculated by solving this matrix equation.

The charge Q_i induced in a specific pickup electrode $i = A, B, C, D$ (cf. Fig. 3) is simply be given by summing up the individual charge densities of the boundary elements which are covered by that

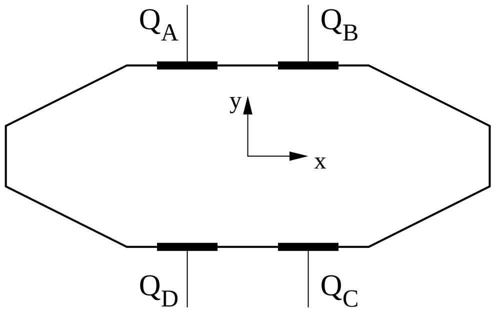

Figure 3: Schematic view of a pickup and definition of the button electrodes.

button electrode. The beam positions are finally received by comparing the induced charges on the four buttons according to the difference over sum (Δ/Σ) method as follows:

$$Q_x = \frac{\Delta_x}{\Sigma} = \frac{(Q_B + Q_C) - (Q_A + Q_D)}{(Q_A + Q_B + Q_C + Q_D)} \tag{19}$$

$$Q_y = \frac{\Delta_y}{\Sigma} = \frac{(Q_A + Q_B) - (Q_C + Q_D)}{(Q_A + Q_B + Q_C + Q_D)} \tag{20}$$

The position sensitivities are determined by the derivatives of Q_x and Q_y with respect to the beam position x_0, y_0:

$$S_x = \frac{\partial Q_x}{\partial x_0}, \qquad S_y = \frac{\partial Q_y}{\partial y_0} \tag{21}$$

The monitor constant which is a characteristic value of an electrostatic monitor is related to the sensitivity by the following formula:

$$k_{x,y} = \frac{1}{S_{x,y}(0,0)} \ . \tag{22}$$

With knowledge of the monitor constant the beam position can be reconstructed from the induced pickup signals in first order by

$$x = k_x \frac{\Delta_x}{\Sigma} = k_x \frac{(Q_B + Q_C) - (Q_A + Q_D)}{(Q_A + Q_B + Q_C + Q_D)} \tag{23}$$

$$y = k_y \frac{\Delta_y}{\Sigma} = k_y \frac{(Q_A + Q_B) - (Q_C + Q_D)}{(Q_A + Q_B + Q_C + Q_D)} \tag{24}$$

3 Results

In PETRA-III it is planned to install five different types of pickup monitors. In the following table their properties are summarized.

Table 1: Properties of the BPMs included to be installed in PETRA-III (status November 2006)

location (number)	chamber profile	size [mm]	ø button [mm]
old octants (109) / straight sections (3)	elliptical	80 × 40	11
new octant (40)	octagon	80 × 38	11
new octant, next to ID (16)	elliptical	66 × 11	11
straight sections (18)	round	ø 94	15
damping wiggler section (24)	racetrack type	120 × 30	11

The position sensitivities has been calculated based on the BEM for these monitor configurations. The scanning range for the beam position (x_0, y_0) was chosen in such a way that it scales linearly with the required measurement range of ± 4 mm for a monitor constant $k_{x,y} = 10$ mm according to the monitor specifications in Ref. [4]. The results of these calculations are summarized in the following subsections.

elliptical profile, old octant and straight sections

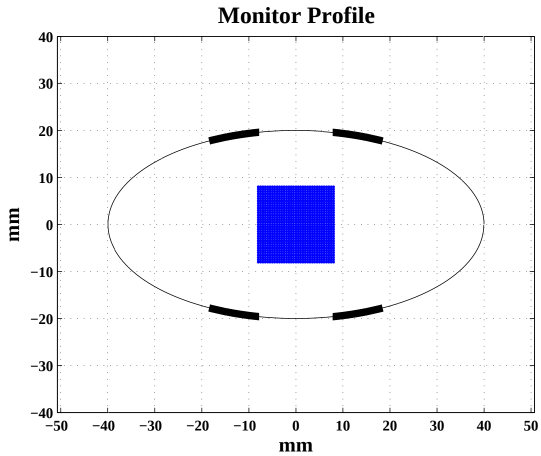

Figure 4: Chamber profile as implemented in the BDE calculation. The blue points correspond to different beam positions for which the reconstructed positions have been calculated according to Eqn.(23,24).

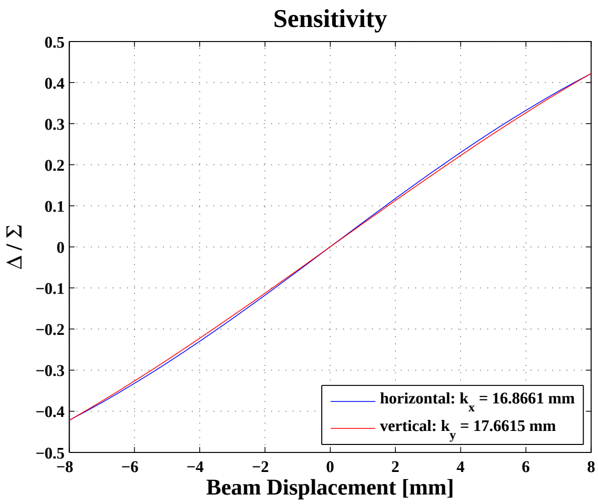

Figure 5: Central horizontal and vertical sensitivity. The corresponding values are $S_x(0{,}0) = 0.0593/\text{mm}$ and $S_y(0{,}0) = 0.0566/\text{mm}$.

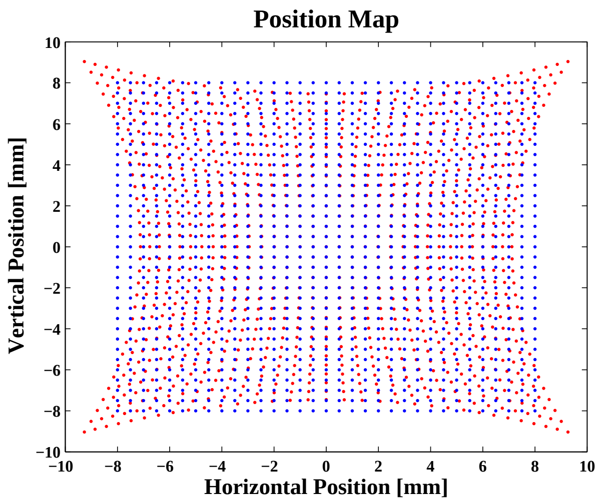

Figure 6: Calculated position map. The blue points indicate the original beam positions, the red ones the reconstructed positions according to Eqn.(23,24).

octagon profile, new octant

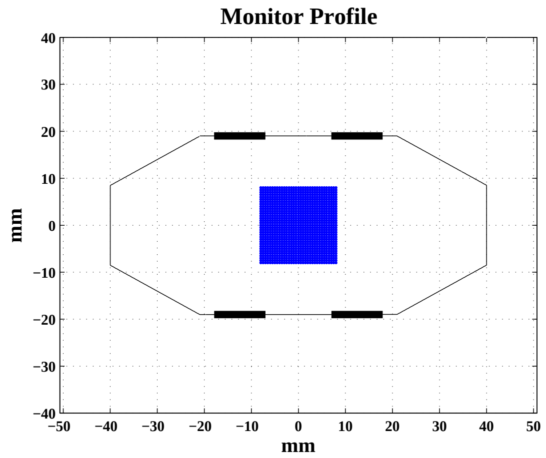

Figure 7: Chamber profile as implemented in the BDE calculation. The blue points correspond to different beam positions for which the reconstructed positions have been calculated according to Eqn.(23,24).

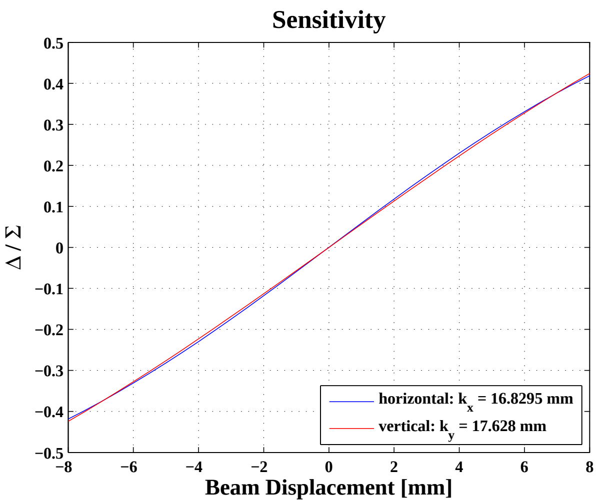

Figure 8: Central horizontal and vertical sensitivity. The corresponding values are $S_x(0,0) = 0.0594/\text{mm}$ and $S_y(0,0) = 0.05567/\text{mm}$.

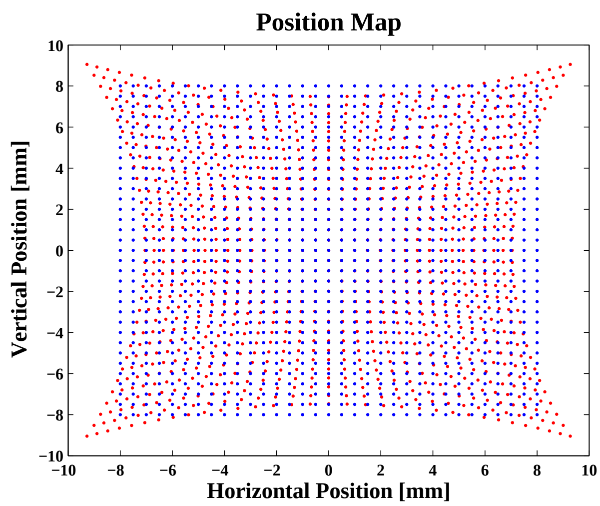

Figure 9: Calculated position map. The blue points indicate the original beam positions, the red ones the reconstructed positions according to Eqn.(23,24).

elliptical profile, next to insertion devices

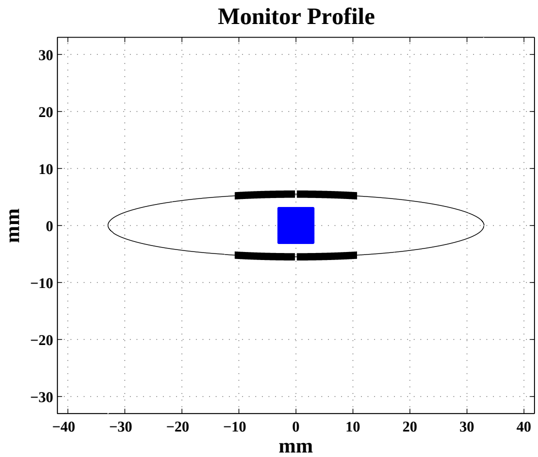

Figure 10: Chamber profile as implemented in the BDE calculation. The blue points correspond to different beam positions for which the reconstructed positions have been calculated according to Eqn.(23,24).

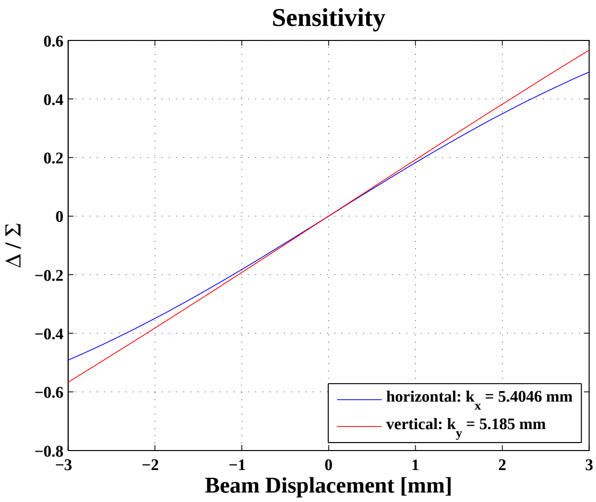

Figure 11: Central horizontal and vertical sensitivity. The corresponding values are $S_x(0{,}0) = 0.1850/\mathrm{mm}$ and $S_y(0{,}0) = 0.1929/\mathrm{mm}$.

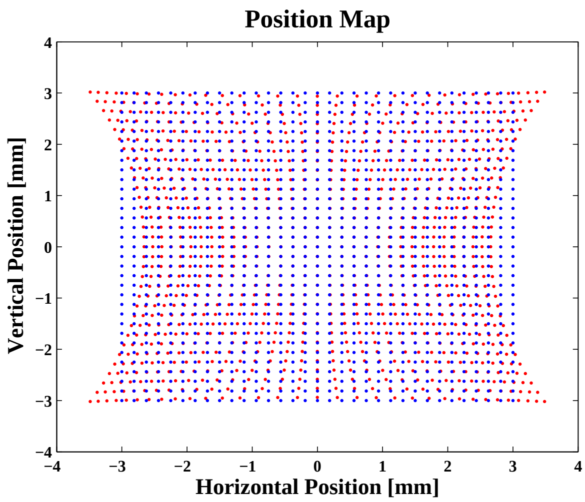

Figure 12: Calculated position map. The blue points indicate the original beam positions, the red ones the reconstructed positions according to Eqn.(23,24).

round profile, straight sections

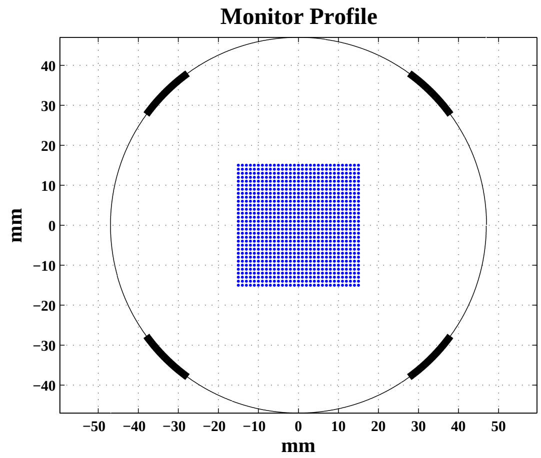

Figure 13: Chamber profile as implemented in the BDE calculation. The blue points correspond to different beam positions for which the reconstructed positions have been calculated according to Eqn.(23,24).

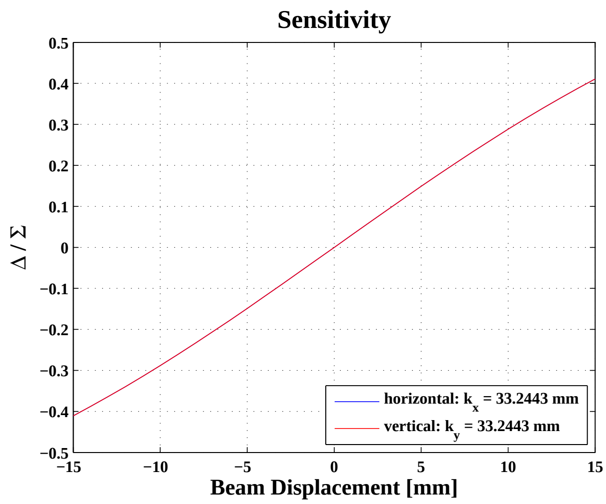

Figure 14: Central horizontal and vertical sensitivity. The corresponding values are $S_{x,y}(0{,}0) = 0.0301/\mathrm{mm}$.

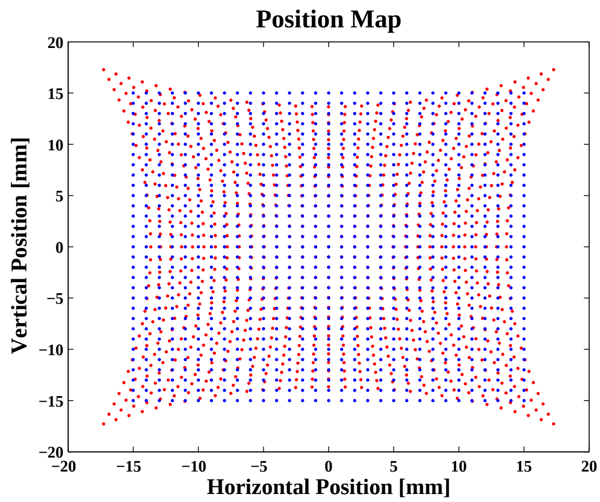

Figure 15: Calculated position map. The blue points indicate the original beam positions, the red ones the reconstructed positions according to Eqn.(23,24).

racetrack type profile, damping wiggler section

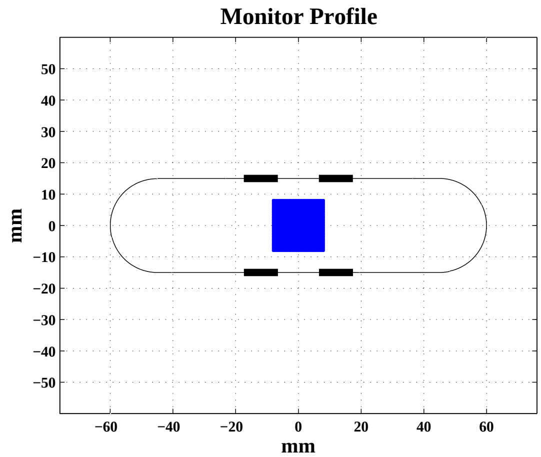

Figure 16: Chamber profile as implemented in the BDE calculation. The blue points correspond to different beam positions for which the reconstructed positions have been calculated according to Eqn.(23,24).

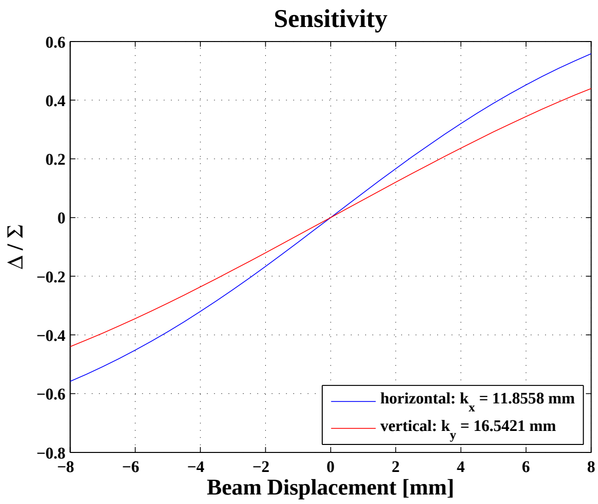

Figure 17: Central horizontal and vertical sensitivity. The corresponding values are $S_x(0{,}0) = 0.0843$/mm and $S_y(0{,}0) = 0.0605$/mm.

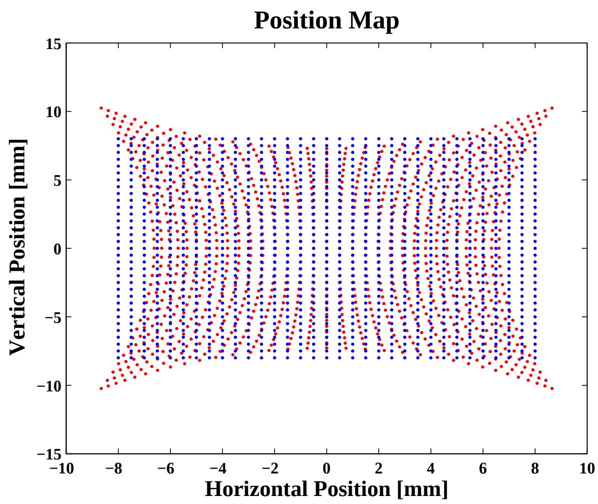

Figure 18: Calculated position map. The blue points indicate the original beam positions, the red ones the reconstructed positions according to Eqn.(23,24).

4 Summary

The position sensitivity for the different electrostatic PETRA-III BPMs has been calculated using a boundary element method. The resulting monitor constants $k_{x,y}$ are in good agreement with independent calculations [4] performed with the computer code MAFIA [5] which is based on Finite Integration Technique (FIT).

In addition for a round beam pipe profile the BEM calculation can be compared directly with an analytical estimate according to the following equation:

$$S_{x,y} = \frac{\sqrt{2}}{R} \frac{\sin \alpha}{\alpha} \, . \tag{25}$$

Here R is the radius of the beam pipe and α the half opening angle of the button electrode viewed from the monitor axis [1]. With the parameters of the PETRA-III monitor for the straight sections the sensitivity can be calculated to $S_{x,y} = 0.0300/\text{mm}$. The corresponding value according to the BEM calculation is 0.0301/mm, cf. Fig. 14.

As result it can be concluded that the bondary element method is a fast and suitable method to calculate the position sensitivity of beam position monitors.

References

[1] T. Shintake et al., Nucl. Instr. Meth. **A 254** (1987) 146

[2] J.H. Cuperus, Nucl. Instr. Meth. **145** (1977) 219

[3] A. Stella, *Analysis of the DAΦNE Beam Position Monitor with a Boundary Element Method*, DAΦNE Technical Note **CD-10** (1997)

[4] I. Krouptchenkow, K. Wittenburg, M. Werner, H.T. Duhme, J. Klute, K. Balewski, R. Neumann, G. Kube, *PETRA III Beam Position Monitor Electronic: Requirements and technical Specifications; Tendering and Contract Management* (2007)

[5] MAFIA. Release 4, Nov 2004. CST GmbH