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Monochromaticity of the Smith–Purcell Optical Radiation Generated by a 75-keV Electron Beam

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The monochromaticity of the Smith–Purcell optical radiation generated by a 75-keV electron beam with a final emittance of $\epsilon = 0.65 \times 10^{-4}$ mm rad that passes over a grating with a period of $D = 0.833$ µm has been analyzed. It has been shown that the monochromaticity of Smith–Purcell radiation is determined not only by the angular aperture of a monochromator but also by the divergence of the electron beam. © *2005 Pleiades Publishing, Inc.*

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Free-electron lasers have recently been created and are successfully used to generate intense monochromatic radiation in the infrared and submillimeter ranges [1]. Such free-electron lasers involve an electron beam with an energy on the order of 50–200 MeV that passes through an undulator with a several-centimeter period. A promising scheme is a free-electron laser based on the Smith–Purcell effect, which will provide radiation in the above range on an electron beam with energy below 1 MeV [2]. Smith–Purcell radiation (SPR) is generated when a charged particle passes parallel to the surface of a periodic structure (e.g., diffraction grating). The mechanism of radiation was predicted by Frank [3] and was experimentally observed for the first time by Smith and Purcell [4] in the optical range on a 300-keV electron beam. The intensity of SPR in the optical range was studied in [5–9] on electron beams with energies from 20 keV [9] to 855 MeV [8]. However, the monochromaticity of the radiation has not yet been investigated experimentally.

For a unidirectional electron beam, the position of the line in the radiation spectrum is expressed in terms of the emission angle and grating period through the known dispersion relation [4]

$$\lambda_n = \frac{D}{|n|}\left(\frac{1}{\beta} - \cos\theta\sin\Phi\right), \qquad (1)$$

where λ is the radiation wavelength, D is the grating period, n is the diffraction order, $\beta = v/c$ is the ratio of the electron velocity to the speed of light, and θ and Φ are the emission angles of SPR photons as shown in Fig. 1.

The monochromaticity of SPR for the unidirectional beam at a fixed emission angle of SPR photons is determined by the number N of grating periods:

$$\Delta\lambda/\lambda \approx 1/N.$$

The angular distribution of radiation energy for the nth order is given by the expression [8]

$$\frac{dW_n}{d\Omega} = 2\pi\alpha\hbar c\frac{n^2}{D}N\frac{\sin^2\theta\sin^2\Phi}{\left(\frac{1}{\beta} - \cos\theta\sin\Phi\right)^3}$$
$$\times |R_n|^2\exp\left(-\frac{4\pi d}{\lambda\beta\gamma}\sqrt{1 + (\beta\gamma\sin\theta\cos\Phi)^2}\right), \qquad (2)$$

where α is the fine structure constant, $|R_n|^2$ is the radiation factor determined by the grating profile, and γ is the Lorentz factor.

The factor $|R_n|^2$ for nonrelativistic electrons is determined by the grating profile and photon emission angles θ and Φ. This quantity is calculated using various models, which give results differing from each other by more than one order of magnitude. For esti-

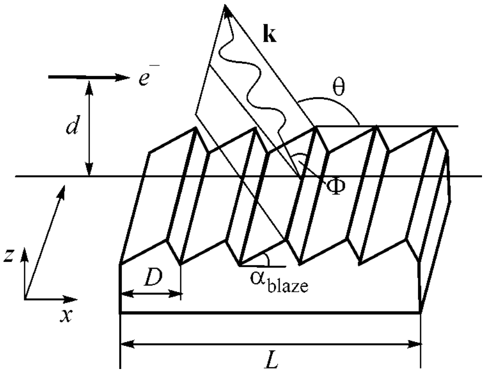

Fig. 1. Scheme of the generation of the Smith–Purcell radiation: **k** is the wavevector, α_{blaze} is the grating-blaze angle, and d is the distance between the electron trajectory and grating (impact parameter).

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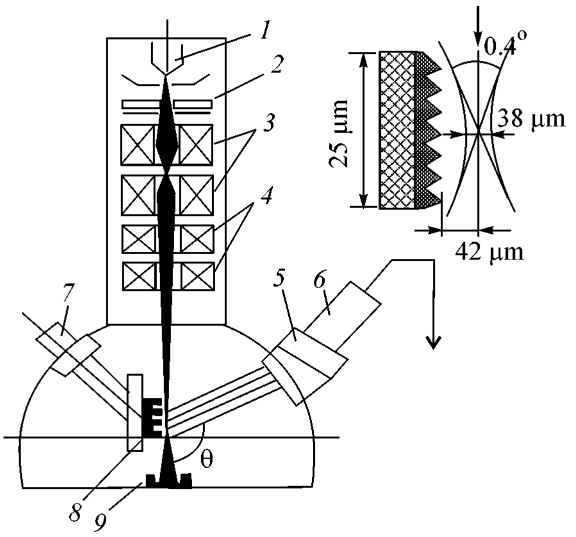

Fig. 2. Geometry of the experiment: (*1*) cathode, (*2*) anode, (*3*) condensers, (*4*) beam-displacement system, (*5*) monochromator, (*6*) photomultiplier tube, (*7*) micrometer screw, (*8*) grating, and (*9*) Faraday cylinder.

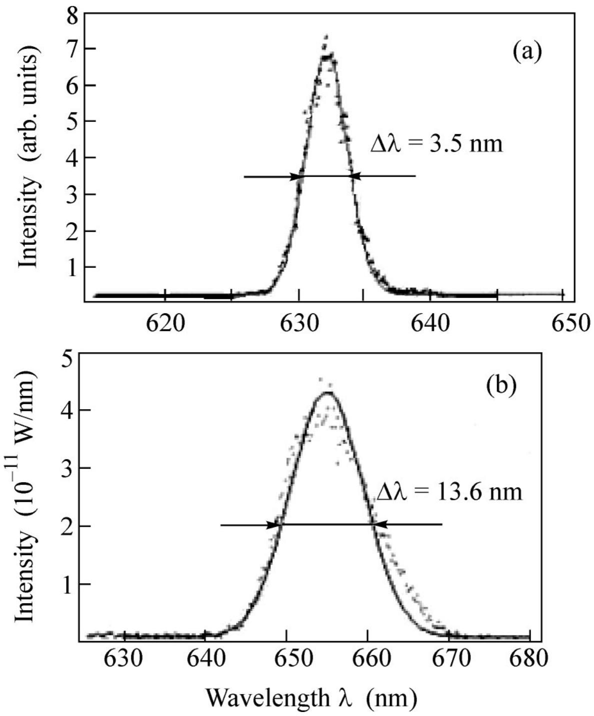

Fig. 3. Emission spectrum of the (a) laser and (b) laser diode.

mates under our experimental conditions, we can take $|R_n|^2 = \text{const} = 1$ according to the Van der Berg model [10]. The intensity of SPR decreases exponentially as the impact parameter d increases. The interaction parameter

$$h_{\text{int}} = \lambda\beta\gamma/4\pi,$$

determines the efficiency of the "coupling" between the beam and grating. In the experiment described in [4], the electron beam size (150 μm) was much larger than the interaction parameter $h_{\text{int}} \approx 0.1$ μm.

The experimental setup for obtaining and studying SPR in the optical range was created on the basis of an EMMA-2 electron microscope. The scheme of the setup is shown in Fig. 2. The electron microscope generates an electron beam with an energy of 25–100 keV and a current of about 1–3 μA. The cross section of the beam in the focus at the diffraction-grating center was $\leq 38 \pm 2$ μm. The distance between the electron beam axis and grating in the focus was approximately equal to 42 μm, which virtually excludes the interaction of the beam particles with the grating. For our experiment, $h_{\text{int}} \approx 0.03$ μm. It is worth noting that the beam diameter in experiments described in [5] and [6] was equal to 200 μm and 5 mm, respectively. The electron energy spread was estimated as $\Delta E/E < 1\%$. The beam emittance, which was measured using the beam sizes at four points and the secondary electron emission current from four movable plates of the charge-sensitive electrometer, was equal to 0.65×10^{-4} mm rad [11]. The plates were moved by means of a micrometer screw, which allowed both the measurement of the beam diameter with a spatial resolution of about 2 μm and the determination of the emittance from a simple geometric construction. The beam divergence was estimated as $\Delta\theta \approx 0.4°$.

A triangular profile grating (see Fig. 1) with a period of 833 nm and an angle of $\alpha_{\text{blaze}} = 26.44°$ was used in the experiment. For the grating length $L = 25$ mm, the number N of periods was approximately equal to 30000. The grating was made of glass and covered by an aluminum coating with a thickness of 700 ± 50 nm. The observation of SPR was performed at an angle of $\theta = 135°$, which corresponds to a radiation wavelength of $\lambda = 745$ nm for $n = 3$ and an electron energy of $E = 75$ keV ($\beta = 0.49$).

The radiation spectrum in the range 500–760 nm was measured in the experiment using an optical monochromator with a photomultiplier tube (PMT) operating in the count regime. The resolution of the monochromator was controlled by the width of the input aperture. For a chosen monochromator aperture width of 2 mm, the resolution was measured using a laser with a wavelength of $\lambda_{\text{las}} = 634$ nm and a linewidth of approximately $\approx 10^{-2}$ nm and was equal to $\Delta\lambda \approx 3.5$ nm ($\Delta\lambda$ is the FWHM of the measured spectrum, see Fig. 3a). To calibrate the optical system, a laser diode ($\lambda = 655$ nm), as well as red ($\lambda = 665$ nm) and blue ($\lambda = 490$ nm) light diodes, is used in addition to the above laser. Figure 3b shows the emission spectrum of the laser diode measured by the monochromator. The laser power was measured by a calorimeter and was equal to 0.37 mW. Two sequential neutral filters attenuate the laser beam by a factor of $k = 1.3 \times 10^{-6}$. For this reason, a PMT with

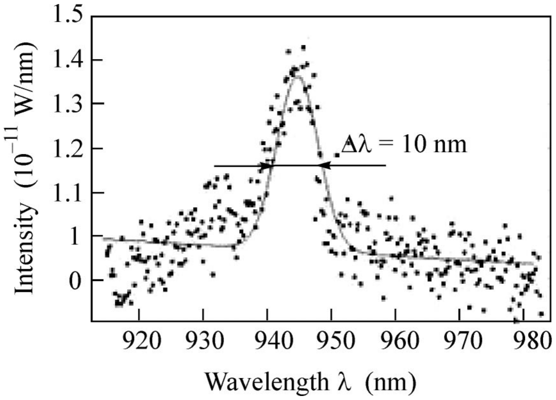

Fig. 4. (Points) Measured spectrum of Smith–Purcell radiation and (solid line) the fit by a Gaussian plus constant background.

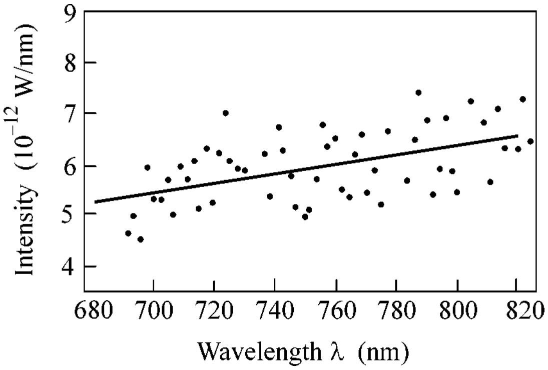

Fig. 5. (Points) Radiation spectrum for the case where the beam touches the grating and (solid straight line) a linear fit.

the same gain as that for the measurement of the SPR spectrum was used as the detector. The measurement results made it possible to obtain the coefficient for recalculation of the number of PMT counts to watts.

For the normal position of the grating ($\Phi = 90°$), we observed a bright peak at a wavelength of $\lambda_3 = 745$ nm, which agrees well with the theory ($\lambda_3^{\text{theor}} = 745.2$ nm), with a width of $\Delta\lambda/\lambda \approx 1.4\%$ (see Fig. 4).

The yield of photons that was calculated taking into account the monochromator aperture and PMT efficiency was approximately equal to $\sim 2 \times 10^{-6}$ photons per electron. For a beam with a current of $I = 1$ μA at an observation angle of $\theta = 130° \pm 1.5°$ and diffraction order $n = 3$, the experimental power of the useful signal (SPR power) was equal to $\Delta P \approx 3.2 \times 10^{-12}$ W.

The solid angle cut by the monochromator aperture and PMT cathode was estimated as $\Delta\Omega = 5 \times 10^{-4}$ in our experiment. Thus, we detected the radiation brightness (the angular distribution of power per unit current, which is proportional to the SPR energy per unit solid angle)

$$\frac{\Delta P}{I\Delta\Omega} = \frac{\Delta W}{\Delta\Omega} = 6.4 \times 10^{-9}\ \text{W}/(\mu\text{A ster})$$

$$= 6.4 \times 10^{-3}\ \text{eV}/(e\ \text{ster})$$

The total measurement error does not exceed 50%. In the experiment described in [5], the measured power of the SPR for the second diffraction order and the beam with a current of 0.25 μA and an energy of 100 keV was equal to $\Delta P \approx 3 \times 10^{-11}$ W and brightness per unit current was

$$\frac{\Delta P}{I\Delta\Omega} = 16 \times 10^{-9}\ \text{W}/(\mu\text{A ster}).$$

An SPR power of 34 μW/(cm^2 ster) was obtained in the optical range for a beam with a current of 3 mA and an energy of 120 keV in the experiment described in [6], where a detector scanned a grating area of 0.5 mm^2, which corresponds to the brightness

$$\frac{\Delta P}{I\Delta\Omega} = 5.7 \times 10^{-11}\ \text{W}/(\mu\text{A ster}).$$

Our result is slightly less than the result obtained in [5], but it is much larger (by approximately two orders of magnitude) than the result obtained in [6].

We carried out an additional experiment in which an electron beam slightly touched the surface in the initial and final sections of the grating, which was accompanied by the observed luminescence of these sections. Figure 5 shows the measured radiation spectrum. In this case, the radiation intensity is much higher, but no lines are observed in the spectrum. Transition radiation with a continuous spectrum is likely generated when the beam touches the grating. The effect of horizontal angular divergence on the position and shape of the SPR line was previously analyzed in [12]. It was shown that the position of the line in the spectrum in this case is determined by the Smith–Purcell formula with the replacement of the grating period D by $D/\cos\Psi$, where Ψ is the angle between the beam-velocity projection onto the grating plane and perpendicular to the "top" of a blaze. In our case, the effects associated with the horizontal divergence of the beam on the lineshape are negligible.

However, the effect of the vertical angular divergence and the finite angular aperture $\Delta\theta$ of the detector on the position and shape of the SPR line is considerable even in the simplest model [13] in which a real grating is represented by a set of ideally conducting strips separated by vacuum gaps. For zero emittance and the aperture $\Delta\theta = 3°$, we estimated the monochromaticity as $\Delta\lambda/\lambda = 0.7\%$. Figure 6 shows the calculation of the SPR line for the out aperture after averaging over the vertical angular distribution, which was approximated by a Gaussian with a variance of $\sigma^2 = 1.96$ mrad^2. According to this figure, the SPR line is

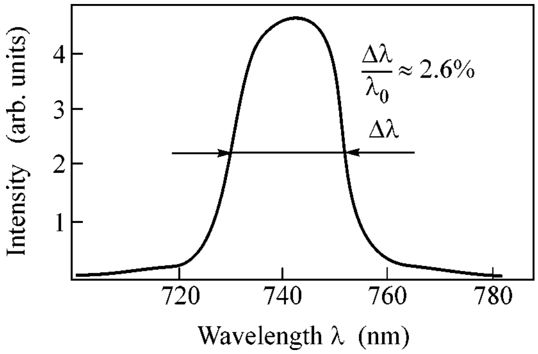

Fig. 6. Shape of the Smith–Purcell radiation line after averaging over vertical emittance for experimental values of the monochromator aperture.

broadened to $\Delta\lambda/\lambda = 2.6\%$. The experiment geometry was chosen such that the particle trajectory touches the grating at its edge for the maximum flight angle $\theta_{0\,\max} = 3\sigma$. The line is broadened due primarily to an increase in the distance between the flying electron and the grating for the nonzero flight angle θ_0. When the electron is spaced from the grating at distances larger than h_{int}, the remaining part of the grating makes almost no contribution; i.e., the SPR line is formed by N_{eff} periods rather than N periods, where the effective number of periods is $N_{\text{eff}} \ll N$ and is determined by the initial section of the grating. Using the experimental linewidth, we estimate the effective number of periods that make a contribution to the SPR line as $N_{\text{eff}} \approx \lambda/\Delta\lambda \approx 70$.

The broadening of the SPR was calculated using the simplest SPR model in which the real grating profile was disregarded. However, this approximation yields a value close to the experimental value, which makes it possible to obtain semiquantitative estimates for the inclusion of the effect of the finite beam emittance on the characteristics of SPR. In turn, this circumstance enables one to use the results to estimate the possibility of creating a new type of free-electron lasers based on SPR [7], where the monochromaticity of the spectral line is determining.

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